Reference frames and notations for NORSUB MRUs

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1 Notations

The NORSUB MRU outputs values (position and velocity) for all 6 degrees of freedom (DOFs). The 3 translational DOFs are surge, sway and heave. The 3 rotational DOFs are roll, pitch and yaw. The notation used for the velocity and position for all DOFs are given by Table 1.

Table 1: Degrees of freedom.

no.	DOF	Linear & angular velocities	Positions & Euler angles
1	Surge	u	x
2	Sway	v	y
3	Heave	w	z
4	Roll	p	ϕ
5	Pitch	q	θ
6	Yaw	r	ψ

Organized in vectors, these are

$$\boldsymbol{p} = \begin{bmatrix} x \\ y \\ z \end{bmatrix}, \quad \boldsymbol{v} = \begin{bmatrix} u \\ v \\ w \end{bmatrix}, \quad \boldsymbol{\Theta} = \begin{bmatrix} \phi \\ \theta \\ \psi \end{bmatrix}, \quad \boldsymbol{\omega} = \begin{bmatrix} p \\ q \\ r \end{bmatrix}, \tag{1}$$

where $\boldsymbol{p} \in \mathbb{R}^{3\times1}$ is the linear position, $\boldsymbol{v} \in \mathbb{R}^{3\times1}$ is the linear velocity, $\boldsymbol{\Theta} \in \mathbb{R}^{3\times1}$ is the angular position, also referred to as attitude in euler angles. $\boldsymbol{p} \in \mathbb{R}^{3\times1}$ is the angular velocity, also called the turn rate vector or rotational velocity.

2 Reference frames

The motion measured by the MRU can be represented in different reference frames. The 3 most important frames are

- NED, $\{n\}$: The North, East, Down frame with axes $\{n\} = [x_n, y_n, z_n]$,
- BODY, $\{b\}$: The body frame with axes $\{b\} = [x_b, y_b, z_b]$,
- MRU, $\{m\}$: The MRU frame with axes $\{m\} = [x_m, y_m, z_m]$.

The NED (North, East, Down) frame is seen in Figure 1. This is a local tangential frame fixed to the Earth's surface.

The MRU frame is the reference frame fixed to the MRU as seen in Figure 2. All accelerometer and gyro raw measurements are performed in this frame.

The vessel's Body frame, $\{b\}$ is the frame fixed to the vessel as seen in Figure 3, which also shows the NED and MRU frame. The MRU and Body frame rotates together as the MRU is mounted to the vessel. The attitude of the MRU with respect to the NED frame is measured by the MRU. The Body frame of a vessel is a right hand coordinate system with x-axis forward and y-axis to starboard.

The MRU can output roll, pitch, yaw, surge, sway and heave motion measurements. The positive directions for these degrees of freedom are given in Figure 4.

An example of an MRU mounted on a ship is seen in Figure 5. The MRU is displayed with its reference frame and the origin of the vessel's Body frame is seen at the aft of the ship. Use the mounting and position wizard in the configuration software to configure the MRU such that the roll, pitch, yaw, surge, sway and heave of the vessel can be measured.

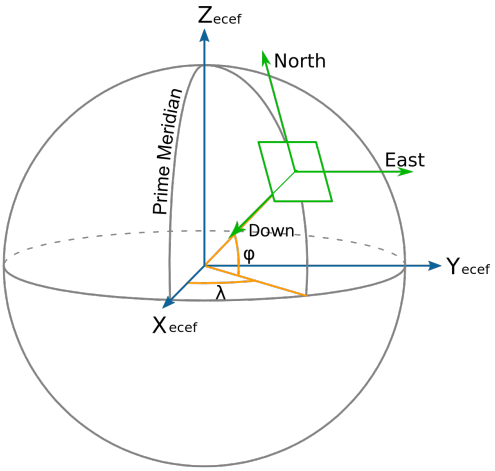

Figure 1: NED frame

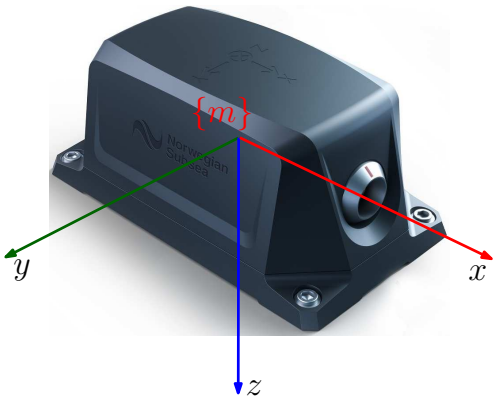

Figure 2: MRU axes

3 Rotation matrix and euler angles

The rotation matrix from $\{b\}$ to $\{n\}$ is $\boldsymbol{R}_b^n(\boldsymbol{\Theta}_{nb}) \in SO(3)$ and is calculated as

$$\boldsymbol{R}_b^n(\boldsymbol{\Theta}_{nb}) = \boldsymbol{R}_{z,\psi}\boldsymbol{R}_{y,\theta}\boldsymbol{R}_{x,\phi}, \tag{2}$$

where

$$\boldsymbol{R}_{x,\phi} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \mathrm{c}\phi & -\mathrm{s}\phi \\ 0 & \mathrm{s}\phi & \mathrm{c}\phi \end{bmatrix}, \; \boldsymbol{R}_{y,\theta} = \begin{bmatrix} \mathrm{c}\theta & 0 & \mathrm{s}\theta \\ 0 & 1 & 0 \\ -\mathrm{s}\theta & 0 & \mathrm{c}\theta \end{bmatrix}, \; \boldsymbol{R}_{z,\psi} = \begin{bmatrix} \mathrm{c}\psi & -\mathrm{s}\psi & 0 \\ \mathrm{s}\psi & \mathrm{c}\psi & 0 \\ 0 & 0 & 1 \end{bmatrix}, \tag{3}$$

are the rotations matrices from roll, pitch and yaw, respectively. c and s are short for cosine and sine. Note that the order of rotation is important.

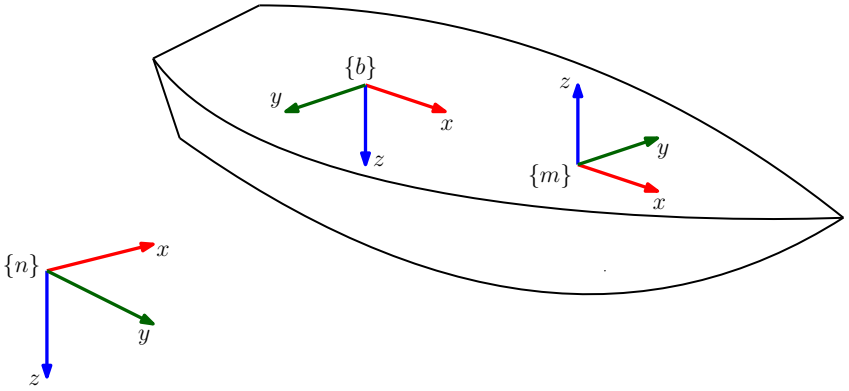

Figure 3: NED, Body (vessel) and MRU frames.

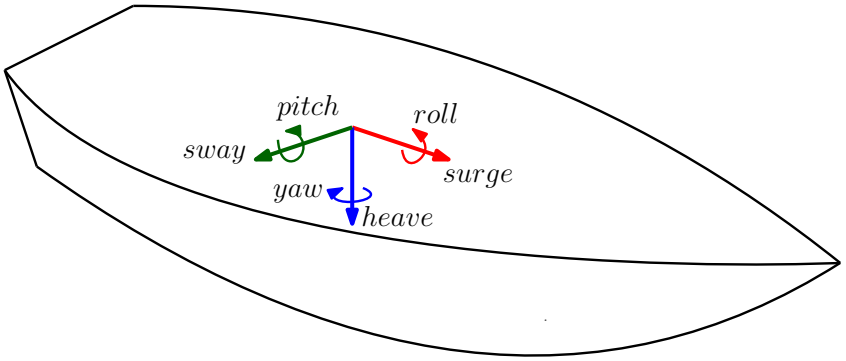

Figure 4: Directions for surge, sway and heave.

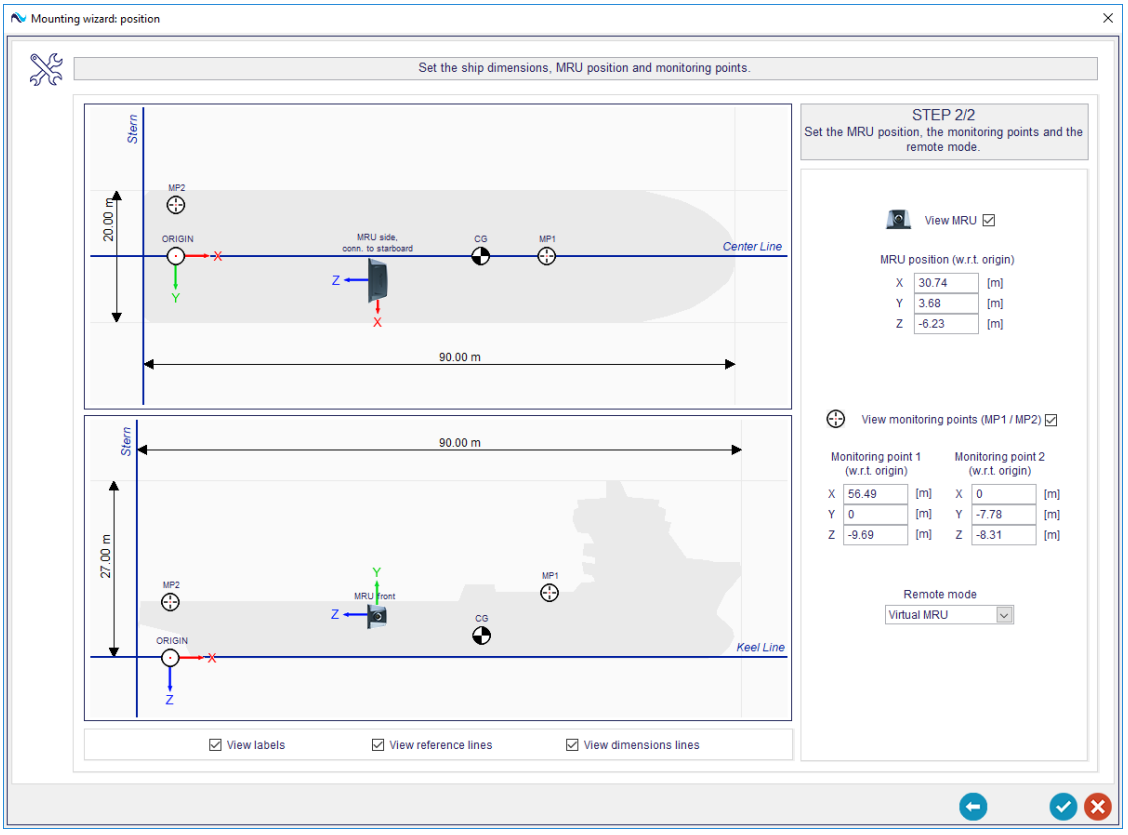

Figure 5: MRU and vessel's body frames.